Applied Mechanics and Materials Vol. 307 (2013) pp 156-160
Online available since 2013/Feb/13 at www.scientific.net

doi:10.4028/www.scientific.net/AMM.307.156

Explicit Formula for Estimating Aerodynamic Drag on Trains Running in Evacuated Tube Transportation

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Keywords: Evacuated Tube Transportation; Train; Aerodynamic drag; Blockage ratio; Explicit formula

Abstract. Because of reducing aerodynamic drag, the maglev train could run at a high-speed in the partial vacuum tube. Scientists of some conutries such as U.S., Swiss and China, have started the research work on high-speed tube trains. In this situation, evacuated tube transportation aerodynamics becomes an important theory research aspect, in which the main study content is how to calculate aerodynamic drag. Based on the explicit formula for estimating aerodynamic drag on moving body in an infinite boundary surroundings put up by Isaac Newton, the evacuated tube surroundings is analyzed and the explicit formula with blockage ratio as an independent variable for estimating aerodynamic drag acted on trains running in the evacuated tube which is a finite space is deduced. With the calculation case, compared with the results came out from the explicit formula got in this paper and the results got by Fluent software, it was found that those results are closed. Thus, the explicit formula created in this paper for conveniently estimating aerodynamic drag based on trains running in evacuated tube transportation is credible.

Introduction

Evacuated Tube Transportation (ETT) will carry out "Space Travel on Earth". Because the aerodynamic drag is eliminated, maglev vehicles could run at ultra-high-speed in ETT tube, for example, over 6000 km/h, and low energy consumption, low noise, low pollution and safer relatively. ETT aerodynamics is one among throng subjects that ETT refers to, and its key mission is to study the aerodynamic drag on ETT vehicles running in the vacuum tube[1-3]. ETT is a new area, short of basic subject theory, and study on aerodynamic drag mainly depends on the research method of rarefied gas dynamics in aviation and the spaceflight field. As for how to calculate the aerodynamic drag, there have been some basic theory such as the mass conservation equation, the momentum conservation equation and the energy conservation equation, concretely example for Euler equation and Navier-Stokes（N-S）equation, usable for reference. However, the vacuum surroundings of ETT is different from the vacuum surroundings of aviation and spaceflight. Aircrafts flight in the infinite boundary surroundings, but ETT maglev trains run in the finite boundary surroundings which is smaller. Therefore, the research background of ETT aerodynamics is different from that of aviation or spaceflight, and the surroundings condition for calculating the aerodynamic drag is different too.

Although it's feasible to use the calculation theory based on Euler equation and Navier-Stokes equation for carrying out simulation analysis and calculation of ETT aerodynamics by changing boundary conditions, such simulation calculating is always complex and it is necessary to use special software and spend a lot of time for finishing it. Anyway, in some cases, simplified analysis and fast calculation are necessary. There have been simplified calculation formula for estimating aerodynamics drag on the moving objects in open surroundings. Based on analyzing ETT surroundings, this paper studies and deduces the simple explicit formula for estimating aerodynamic drag on ETT vehicles.

1. Classical aerodynamic drag explicit formula

According to the educing and conclusion by Isaac Newton, the aerodynamic drag acted on moving body depends on the density of the air, the square of the velocity, the size and shape of the body as following equation (1) [4-6].

$$F_D = \frac{1}{2} \cdot C_D \cdot \rho \cdot A \cdot v^2 \tag{1}$$

Where F_D is the drag acted on the moving body; C_D is the drag coefficient which depends on body shape, inclination, air viscosity and compressibility; ρ is the density of the air; A is the section area of the moving body; v is the velocity of the moving body.

It should be noticed that equation (1) is created for the moving body in an open environment, namely body in a surroundings with an infinite boundary, and the air could freely diffuse when the moving body oppress the air. However, for the train running in the vacuum tube (low pressure or rarefied gas surroundings), the environment is closed and the boundary is finite. The moving train in ETT tube would press air, but the air couldn't completely freely diffuse for being restricted by the tube wall. Therefore, equation (1) doesn't fit to solving aerodynamic drag acted on a running train in ETT tube.

Although we may simulate and calculate the air drag acted on trains running in ETT tube by some fluid calculation software, fast simple explicit calculation means is desired by people. Thus, if we could improve equation (1) successfully, and develop it into explicit format fitting to calculating aerodynamic drag acted on moving body in a closed surroundings like ETT tube, it will be an important achievement of ETT aerodynamics.

2. General form of air drag formula with the tube diameter d as variable

For the ETT where the train figure has been confirmed, drag coefficient C_D and section area A in equation (1) are the known constant. The velocity v is a variable, but it doesn't change with boundary conditions. Thus, only the air density ρ changes when the infinite boundary for which the equation (1) is fitting is transferred into the finite boundary of ETT. We only need to study variety of ρ, seek for the variables leading to ρ change and put up function relationship leading to ρ change, then we can create the new drag equation which fits to running trains in a finite boundary surroundings----ETT tube.

In fact, when the infinite boundary surroundings where equation (1) fits to is changed into the finite boundary of ETT, only the tube diameter d changes, namely d changes from infinite size to a finite size. Thus ρ is the function of d, $\rho = \rho(d)$. Then we can get the general form of equation (1), as equation (2), which fits to both infinite boundary surroundings and finite boundary surroundings.

$$F_D = F_D(d) = \frac{1}{2} \cdot C_D \cdot \rho(d) \cdot A \cdot v^2 \tag{2}$$

The length of ETT tube can be treated as infinite long tube, but the moving train leads to air density change only in a finite length area where the train passes through. We call this area as the impact region. The impact ranges at front and tail are different. As shown in fig. 1, l_f is the impact range at the train front, l_a is the impact range at the train trail, l_{fa} is the total impact range length.

$$l_{fa} = l_f + l_v + l_a \tag{3}$$

Where l_v is the length of the train.

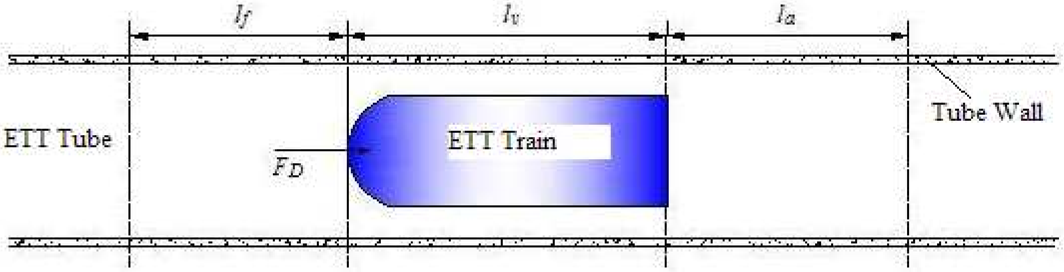

Fig. 1 Aerodynamic impact area of train running in ETT tube

For equation (2) as an estimating formula, we can decide the air density $\rho(d)$ according to the impact region.

Different from the open infinite boundary surroundings, when trains run in the ETT tube, the volume of the train occupy space and press the air in the impact area, making air density in the region change----density increasing. In the case, the density ρ could be expressed by equation (4).

$$\rho = \rho(d) = \frac{\rho_0 \cdot A_t \cdot l_{fa}}{A_t \cdot l_{fa} - A \cdot l_v} = \frac{\rho_0 \cdot \frac{1}{4}\pi d^2 \cdot l_{fa}}{\frac{1}{4}\pi d^2 \cdot l_{fa} - A \cdot l_v} \tag{4}$$

Where ρ_0 is the original air density (vacuum or low pressure degree) in ETT tube, A_t is the section area of ETT tube and d is the inside diameter of the tube. Replace ρ in equation (2) with equation (4), then equation (2) becomes equation (5) as follows.

$$F_D = \frac{1}{2} C_D \cdot \frac{\rho_0 \cdot \frac{1}{4}\pi d^2 \cdot l_{fa}}{\frac{1}{4}\pi d^2 \cdot l_{fa} - A \cdot l_v} \cdot A \cdot v^2 \tag{5}$$

Where the impact region length l_f and l_a will be decided by experiment.

3. Air drag formula with the blockage ratio as variable

When the impact length l_{fa} is infinite long, equation (5) becomes $F_D|_{l_{fa}=\infty} = \frac{1}{2} C_D \cdot \rho_0 \cdot A \cdot v^2$. It's become the express form of equation (1). When the impact area l_{fa} length equals to l_v, the length of train, namely $l_f = 0$, $l_a = 0$, then

$$F_D|_{l_{fa}=l_v} = \frac{1}{2} C_D \cdot \frac{\rho_0 \cdot \frac{1}{4}\pi d^2}{\frac{1}{4}\pi d^2 - A} \cdot A \cdot v^2 = \frac{1}{2} C_D \cdot \frac{\rho_0}{1 - \frac{A}{A_t}} \cdot A \cdot v^2 = \frac{1}{2} C_D \cdot \frac{\rho_0}{1-\alpha} \cdot A \cdot v^2 \tag{6}$$

Where α is the ratio of train section area to tube section area, namely the blockage ratio. In fact, when trains run in ETT tube, it's always $l_f > 0$ and $l_a > 0$, namely $l_{fa} > l_v$. Therefore, the drag value calculated out by equation (6) would be the maximum, namely the real drag value will be always less than the value calculated by equation (6).

Equation (1) expresses the air drag acted on moving body in an infinite boundary surroundings, which is the minimum of the issue. Equation (6) expresses the air drag acted on moving body in a finite boundary surroundings, which is the maximum of the issue. It means the real air drag acted on the train running in ETT tube would be between this maximum and this minimum, thus the corresponding coefficient will be in [1, $\frac{1}{1-\alpha}$]. There are many expressions that meet the condition. Here taking $\beta=(1+\alpha)^x$, when $\alpha\in(0,1)$ and $x\in(0,\infty)$, then $\beta\in[1,\frac{1}{1-\alpha}]$. Where the value of x can be decided by experiment or numerical value analyzing. Then equation (6) could become another form as equation (7).

$$F_D=\frac{1}{2}C_D\cdot\rho_0\cdot\beta\cdot A\cdot\upsilon^2=\frac{1}{2}C_D\cdot(1+\alpha)^x\cdot\rho_0\cdot A\cdot\upsilon^2 \tag{7}$$

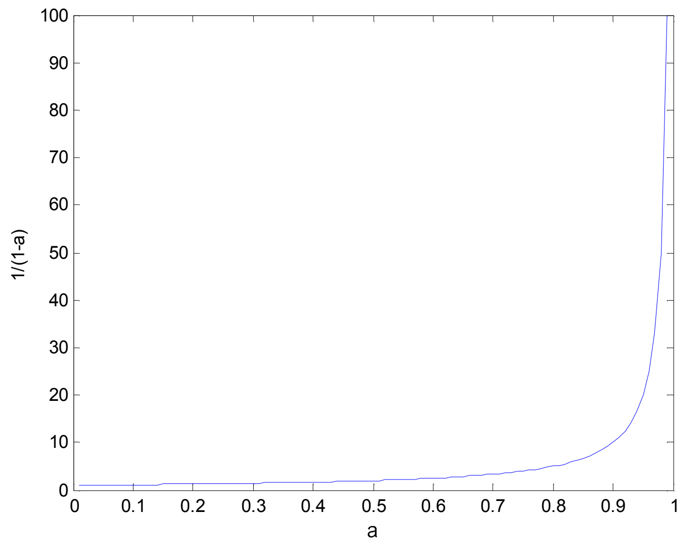

Fig. 2 Relationship curve where the coefficient term $1/(1-\alpha)$ changes with the variable α

It could be seen in Fig. 2, when $\alpha>0.9$, the value of $1/(1-\alpha)$ would hoik, meaning the air drag on the running train in ETT tube would rise sharply when the blockage ratio is more than 0.9. Therefore, the blockage ratio should be in a range less than 0.9.

4. Example calculating

Calculating model: inside tube diameter $d=3$ m, section area of the vehicle $A=3.14$ m^2(the vehicle diameter 2m), hemicycle vehicle head, blockage ratio $\alpha=0.4444$, the drag coefficient $C_D=0.5$.

In turn taking the vacuum in the tube 1/10atm, 1/100atm, 1/1000atm and 1/10000atm, namely the air density $\rho_0=(0.1225,0.01225,0.001225,0.0001225)$ (kg/m^3), according to equation (6), we can calculate out the value of drag acted on the face of the train running at difference speeds as shown in Table 1.

Table 1 The value of the air drag calculated by equation (6) (unit: kN)

ρ_0 (kg/m^3) / υ (m/s)	0.1225	0.01225	0.001225	*0.0001225*
150	2.1638	0.2163	0.0218	*0.0023*
200	3.8465	0.3848	0.0385	*0.0038*
250	6.0103	0.6010	0.0600	*0.0060*
300	*8.6548*	*0.8655*	*0.0865*	*0.0088*

In order to check the reliability of the calculating results by equation (6), let's further process the simulation calculating with the software Fluent 6.3. The calculating model is as the above, other parameters needed for Fluent simulation are vehicle length L=6m, account fluid field region length 66m (as shown in Fig. 3), 2 dimension model, taking half of the whole fluid field as calculation region, the gas stable, incompressible and viscous.

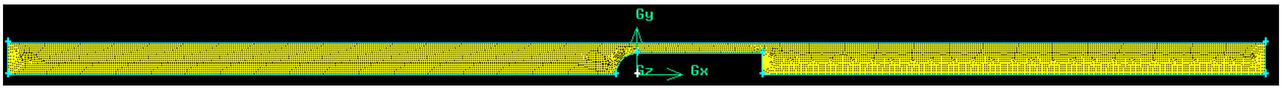

Fig. 3 Calculation fluid region and meshing

By calculating, air drag acted on the train running in ETT tube at different speed and difference air density are as shown in Table 2.

Table 2 The value of the air drag calculated by simulation software Fluent 6.3 (unit: kN)

ρ_0 (kg/m^3) / υ (m/s)	0.1225	0.01225	0.001225	0.0001225
150	2.789	0.251	0.021	0.001
200	5.035	0.455	0.038	0.002
250	7.984	0.720	0.061	0.004
300	11.618	1.047	0.089	0.006

Comparing these air drag values in Table 1 and Table 2, we can see that the results calculated out by the explicit formula (6) are closed to the results by the simulation software. It makes know that the explicit formula (6) is credible and could be taken as the basic formula to estimate air drag acted on the train running in ETT tube for some confirmed working conditions such as the blockage ratio $\alpha = 0.4444$ and the drag coefficient C_D=0.5.

When the blockage ratio $\alpha = 0.4444$, it means that the $x = 1.6$ in the coefficient function $\beta = (1+\alpha)^x$. For other working conditions, the value of x would be variable.

5. Conclusion

ETT aerodynamics is a new branch area of the aerodynamics, also is one of the important research directions of ETT. The main mission of ETT aerodynamics is to study air drag acted on the train running in ETT tube. Anyway, fast estimation equation, especially the explicit formula, are quite necessary for the future ETT research work and engineering application. Based on the finite boundary condition, this paper educed and created the explicit formula for calculating air drag acted on the train running in ETT tube. In the formula (6), blockage ratio is the variable. It is the development and makeup for Newton air drag formula.

By example calculating, it is indicated it is credible to calculate air drag in some working conditions in ETT tube by formula (6).

As for the more general formula (7), it is necessary to get the value of x by further experiments or numerical analysing.

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Mechatronics and Computational Mechanics
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